

# Dual parameterization of generalized parton distributions and a description of DVCS data

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**Abstract.** We discuss a new leading-order parameterization of generalized parton distributions of the proton, which is based on the idea of duality. In its minimal version, the parameterization is defined by the usual quark singlet parton distributions and the form factors of the energy-momentum tensor. We demonstrate that our parameterization describes very well the absolute value, the  $Q^2$ -dependence and the  $W$ -dependence of HERA data on the total DVCS cross section and contains no free parameters in the HERA kinematics. The parameterization suits the low- $x_{Bj}$  region especially well, which allows us to advocate it as a better alternative to the frequently used double distribution parameterization of the GPDs.

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## 1 Introduction

Generalized parton distributions (GPDs) have become a standard QCD tool for analyzing and parameterizing the non-perturbative parton structure of hadronic targets, for reviews see [1–6]. In general, GPDs are more general and complex objects than structure functions and form factors. In addition, experimentally measured observables do not access the GPDs themselves but only their convolution with hard scattering coefficients. Therefore, the experimental determination of GPDs is an extremely difficult task. Hence, when dealing with GPDs, one invariably uses models, the known limiting behavior and general properties of GPDs and the physical intuition.

GPDs have been modeled using virtually all known models of the nucleon structure: bag models [7], the chiral quark-soliton model [35], light-front models [9, 10], constituent quark models [11], the Bethe–Salpeter approach [12], and a NJL model [13]. In addition, a double distribution model of GPDs [14, 15] and modeling by perturbative diagrams [16] have been suggested.

The factorization theorem for deeply virtual Compton scattering (DVCS) [17] gives a practical possibility to measure GPDs by studying various processes involving GPDs: DVCS, exclusive electroproduction of vector mesons, wide angle Compton scattering [18, 19], exclusive  $p\bar{p} \rightarrow \gamma\gamma$  annihilation [20, 21], the  $p\bar{p} \rightarrow \gamma\pi^0$  process [22],  $\gamma^*\gamma \rightarrow \pi\pi$  near threshold [23]. However, in order to accommodate such a potentially large number of data, parameterizations of GPDs should be sufficiently flexible and

versatile. In particular, they should allow for the connection of DVCS with the  $p\bar{p} \rightarrow \gamma\gamma$  process.

The commonly used double distribution parameterization of GPDs [14, 15] is one example of the model of GPDs that could be used to connect different physical channels [24]. However, the parameterization of the GPDs based on the double distribution has several unsatisfactory features. First, in order to have the full form of polynomiality, the so-called  $D$ -term [25] has to be added by hand. Second, in order to describe the low- $x$  HERA data on the total DVCS cross section one has to assume a very specific shape for the input GPDs, which appears unnatural because DVCS asymmetries are described using a rather different shape of GPDs [26]. Third, the model does not allow for an intuitive physical motivation and interpretation, see [27] for a discussion of the physics of GPDs.

In this paper, we offer a new model for GPDs, which was introduced in a general form in [28]. Unlike the models of the GPDs mentioned above, the present model has a simple physical interpretation and direct correspondence to the mechanical properties of the target [29]. The suggested parameterization fulfills the polynomiality condition and also allows for flexible modeling of the  $t$ -dependence of the GPDs, which we shall address in a separate publication.

The considered parameterization of GPDs is called dual because GPDs are presented as an infinite series of  $t$ -channel exchanges, which reminds us of the ideas of duality in hadron-hadron scattering.

In this work, we formulate the minimal version of dual parameterization and determine the free parameters of the model. Using the resulting dual parameterization of GPDs,

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we successfully describe the HERA data on the DVCS cross section [30–32]. We explain that our parameterization suits the low- $x_{Bj}$  kinematics especially well, because the quark singlet parton distributions are never probed at the unmeasurably low values of Bjorken  $x$  and because the final expression for the DVCS amplitude is numerically stable. Thus, the dual parameterization of GPDs gives an opportunity to have a physically intuitive model of GPDs, which agrees with the DVCS experiments and can serve as an alternative to the popular double distribution model.

## 2 The dual parameterization of GPDs

The dual representation of GPDs was first introduced for pion GPDs in [33]. The essence of that derivation is presented below. The starting point was the decomposition of the two-pion distribution amplitude ( $2\pi$ DA) in terms of the eigenfunctions of the ERBL evolution equation (Gegenbauer polynomials  $C_n^{3/2}$ ), the partial waves of produced pions (Legendre polynomials  $P_l$ ) and generalized form factors  $B_{nl}$ . The moments of the  $2\pi$ DA, being the matrix elements of certain local operators, could be related by crossing to the moments of pion GPDs. Then, pion GPDs could be formally reconstructed using the explicit form of their moments.

Based on the result of [33], the dual representation for proton GPDs was suggested in [28]. In this paper, we will consider only the singlet ( $C$ -even) combination of GPDs  $H$ , which give the dominant contribution to the total DVCS cross section at high energies and small  $t$ . We will work within the leading order approximation and, hence, we will consider only quark GPDs.

The dual representation of the singlet GPD  $H^i$  of the quark flavor  $i$  is [28]

$$H^i(x, \xi, t, \mu^2) = \sum_{n=1, \text{ odd}}^{\infty} \sum_{l=0, \text{ even}}^{n+1} B_{nl}^i(t, \mu^2) \theta\left(1 - \frac{x^2}{\xi^2}\right) \times \left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right), \quad (1)$$

where  $x$ ,  $\xi$  and  $t$  are the usual GPD variables. The series (1) is divergent at fixed  $x$  and  $\xi$ , and, hence, it should be understood as a formal series. In particular, it is incorrect to evaluate the series term by term. As a result, the GPD  $H^i$  of (1) has a support over the entire  $-1 \leq x \leq 1$  region, regardless of the fact that each term of the series is non-vanishing only for  $-\xi \leq x \leq \xi$ . The formal representation (1) can be equivalently rewritten as a converging series using the technique developed in [28].

The derivation of (1) used the idea of the duality of hadronic physics, when the scattering amplitude in the  $s$ -channel is represented as an infinite series of the  $t$ -channel exchanges. This explains the name “dual representation” for the representation of (1).

As a double series, (1) is inconvenient for phenomenological applications. For the evaluation of the LO DVCS amplitude, it is useful to introduce the functions  $Q_k(x, t)$

whose Mellin moments generate the  $B_{nl}^i$  form factors [28]

$$B_{n n+1-k}^i(t, \mu^2) = \int_0^1 dx x^n Q_k^i(x, t, \mu^2), \quad (2)$$

where  $k$  is even. A remarkable property of the dual representation is that the  $\mu^2$ -evolution of the functions  $Q_k^i$  is given by the usual leading order (LO) DGLAP evolution.

Since the  $B_{nl}^i$  form factors are related to the moments of  $H^i$ , the  $Q_k$  functions are also constrained by these moments. In particular, from

$$\begin{aligned} \int_{-1}^1 dx x H^i(x, \xi, t) &= M_2^i(t) + \frac{4}{5} \xi^2 d^i(t) \\ &= \frac{6}{5} \left[ B_{12}(t) - \frac{1}{3} (B_{12}(t) - 2B_{10}(t)) \xi^2 \right], \end{aligned} \quad (3)$$

it follows that

$$\begin{aligned} \int_0^1 dx x Q_0^i(x, t, \mu^2) &= \frac{5}{6} M_2^i(t, \mu^2), \\ \int_0^1 dx x Q_2^i(x, t, \mu^2) &= \frac{5}{12} M_2^i(t, \mu^2) + d^i(t, \mu^2), \end{aligned} \quad (4)$$

where  $M_2^i$  at  $t = 0$  is the proton light-cone momentum fraction carried by the quarks, and  $d^i(t)$  is the first moment of the quark  $D$ -term.

In addition, the  $B_{nn+1}$  form factors at the zero momentum transfer are fixed by the Mellin moments of the quark singlet parton distribution functions (PDFs). In particular,

$$\begin{aligned} \frac{3}{4} \int_0^1 dx (q^i(x, \mu^2) + \bar{q}^i(x, \mu^2)) &= B_{10}(0) \\ &= \int_0^1 dx Q_0^i(x, 0, \mu^2). \end{aligned} \quad (5)$$

The  $Q_0^i$  functions at  $t = 0$  are completely fixed in terms of the forward proton PDFs [28]

$$\begin{aligned} Q_0^i(x, 0, \mu^2) &= q^i(x, \mu^2) + \bar{q}^i(x, \mu^2) \\ &\quad - \frac{x}{2} \int_x^1 \frac{dz}{z^2} (q^i(z, \mu^2) + \bar{q}^i(z, \mu^2)). \end{aligned} \quad (6)$$

As suggested in [28], keeping only the functions  $Q_0^i$  and  $Q_2^i$  constitutes the minimal version of the dual parameterization of GPDs. The functions  $Q_0^i$  and  $Q_2^i$  are defined by (6) and (4), where  $M_2^i(t)$  and  $d^i(t)$  have a clear physical interpretation, since they are the form factors of the energy-momentum tensor evaluated between the states representing the given target. At  $t = 0$ ,  $M_2^i(0)$  is the fraction of the plus-momentum of the nucleon carried by the quarks of flavor  $i$ ;  $d^i(0)$  characterizes the shear forces experienced by the quarks in the target.

Next we discuss the minimal version of the dual representation in detail. While  $Q_0^i$  at  $t = 0$  is defined by (6), only the first  $x$ -moment of  $Q_2^i$  is constrained. We simply assume that  $Q_2^i \propto Q_0^i$  and take

$$Q_2^i(x, 0, \mu^2) = \beta^i Q_0^i(x, 0, \mu^2), \quad (7)$$

where  $\beta^i$  are constants. From (4), we obtain

$$\beta^i = \frac{6}{5} \frac{d^i(0)}{M_2^i(0)} + \frac{1}{2}, \quad (8)$$

which gives

$$\beta^u = -4.4, \quad \beta^d = -8.9, \quad \beta^s = 0.5. \quad (9)$$

In this numerical estimate, we assumed that  $d^u = d^d \approx -2$  and  $d^s \approx 0$  at the low normalization point, which is consistent with the estimate of the nucleon  $D$ -term [34],  $\sum_i d^i(0) \approx -4$ , based on the original calculation in the chiral quark soliton model [35]. The uncertainty of this estimate was discussed in [36]. The momentum fractions  $M_2^i$  were evaluated at  $\mu_0 = 0.6$  GeV using LO GRV parton PDFs [37].

In general,  $\beta^i$  depend on  $\mu^2$  because of the dependence of  $d^i$  and  $M_2^i$  on  $\mu^2$ . However, as will be seen from the general expression for the DVCS amplitude, at small values of  $\xi$  typical for HERA data on the total DVCS cross section, the contribution of the  $Q_2^i$  function is kinematically suppressed. Therefore, the goodness of the description of the data is not affected by the exact values of  $\beta^i$ , and we simply used (9) at all  $\mu^2$ .

Until recently, the  $t$ -dependence of the DVCS cross section was not measured. One would simply assume that the DVCS cross section exponentially depends on  $t$ ,

$$\frac{d\sigma_{\text{DVCS}}(x_{Bj}, Q^2, t)}{dt} = e^{-B|t|} \left( \frac{d\sigma_{\text{DVCS}}(x_{Bj}, Q^2, t)}{dt} \right)_{t=0}, \quad (10)$$

such that the total DVCS cross section is

$$\sigma_{\text{DVCS}}(x_{Bj}, Q^2) = \frac{1}{B} \left( \frac{d\sigma_{\text{DVCS}}(x_{Bj}, Q^2, t)}{dt} \right)_{t=0}. \quad (11)$$

The value of the slope parameter  $B$  was rather uncertain,  $5 \leq B \leq 9$  GeV $^{-2}$ . The range of the values covers the experimentally measured range of the  $t$ -slope of electroproduction of light vector mesons at HERA. However, very recently, the  $t$ -dependence of the total DVCS cross section for  $0.1 \leq |t| \leq 0.8$  GeV $^2$  and at  $Q^2 = 8$  GeV $^2$  was measured by the H1 collaboration at HERA and was fitted by the exponential form of (10), with the result  $B = 6.02 \pm 0.35 \pm 0.39$  GeV $^{-2}$  [32].

In our numerical estimates of the DVCS cross section, we calculate the DVCS amplitude at  $t = 0$  and then use (11) in order to find the  $t$ -integrated DVCS cross section. In general, the slope  $B$  should decrease with increasing  $Q^2$ . A particular model for the  $Q^2$ -dependent slope was suggested in [39]:  $B(Q^2) = 8(1 - 0.15 \ln(Q^2/2))$  GeV $^{-2}$ . In our analysis, we use the same  $Q^2$ -dependence,

$$B(Q^2) = 7.6 (1 - 0.15 \ln(Q^2/2)) \text{ GeV}^{-2}, \quad (12)$$

but with a slightly smaller constant 7.6 GeV $^{-2}$ , which is chosen such that (12) reproduces the H1 value of the slope at  $Q^2 = 8$  GeV $^2$ .

In summary, our parameterization of the GPDs  $H^i$  is defined by (4), (6) and (7). The  $t$ -dependence of the DVCS cross section is given by (10) and (12). This is the minimal version of the dual representation of the GPDs, which can be readily extended by considering more  $Q_k^i$  functions, a more elaborate  $t$ -dependence and by taking into account the other GPDs of the proton. The main practical advantage of our representation is that the  $\mu^2$ -evolution of  $Q_{0,2}^i$  is given by the usual DGLAP evolution of the singlet PDFs, see (6).

### 3 Description of low- $x$ HERA data on the DVCS cross section

In this section, we evaluate the total DVCS cross section using the minimal model of the dual representation of GPDs and compare the results to HERA data [31, 32].

The total unpolarized DVCS cross section on the photon level reads, see e.g. [38],

$$\sigma_{\text{DVCS}}(x_{Bj}, Q^2) = \frac{\alpha_{e.m.}^2 x_{Bj}^2 \pi (1 - \xi^2)}{Q^4 \sqrt{1 + 4x_{Bj}^2 m_N^2 / Q^2}} \times \int_{t_{\min}}^{t_{\max}} dt |\bar{A}_{\text{DVCS}}(\xi, t, Q^2)|^2, \quad (13)$$

where  $\alpha_{e.m.}$  is the fine-structure constant,  $\xi = 1/2x_{Bj}/(1 - x_{Bj}/2)$  is the Bjorken limit, and  $t_{\max} \approx 0$  and  $t_{\min} \approx -1$  GeV $^{-2}$ ;  $|\bar{A}_{\text{DVCS}}|^2$  is the squared and spin-averaged DVCS amplitude.

To the leading order in  $\alpha_s$ , the DVCS amplitude is expressed in terms of the singlet combination of the GPDs  $H^i$ ,

$$\mathcal{A}_{\text{DVCS}}(\xi, t, Q^2) = \sum_i e_i^2 \int_0^1 dx H^i(x, \xi, t, Q^2) \times \left( \frac{1}{x - \xi + i0} + \frac{1}{x + \xi - i0} \right). \quad (14)$$

Using our model for GPDs and the results of [28], the DVCS amplitude can be presented in a compact form in terms of the  $Q_0^i$  and  $Q_2^i$  functions

$$\mathcal{A}_{\text{DVCS}}(\xi, t, Q^2) = - \sum_i e_i^2 \int_0^1 \frac{dx}{x} \sum_{k=0}^2 x^k Q_k^i(x, t, Q^2) \times \left( \frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2\delta_{k0} \right). \quad (15)$$

Using the exponential ansatz for the  $t$ -dependence of the DVCS cross section, the total DVCS cross section is expressed in terms of the DVCS amplitude at  $t = 0$  [see (11)]

$$\sigma_{\text{DVCS}}(x_{Bj}, Q^2) = \frac{\alpha_{e.m.}^2 x_{Bj}^2 \pi (1 - \xi^2)}{Q^4 \sqrt{1 + 4x_{Bj}^2 m_N^2 / Q^2}} \frac{1}{B(Q^2)} \times |\bar{A}_{\text{DVCS}}(\xi, t = 0, Q^2)|^2, \quad (16)$$

where  $\mathcal{A}_{\text{DVCS}}(\xi, t = 0, Q^2)$  is given by (15) evaluated with  $Q_{0,2}^i(x, 0, Q^2)$ .

Our predictions for the  $Q^2$ -dependence and  $W$ -dependence of the total DVCS cross section are presented in Figs. 1 and 2, respectively. For comparison, we also present the H1 [32] and ZEUS [31] data.

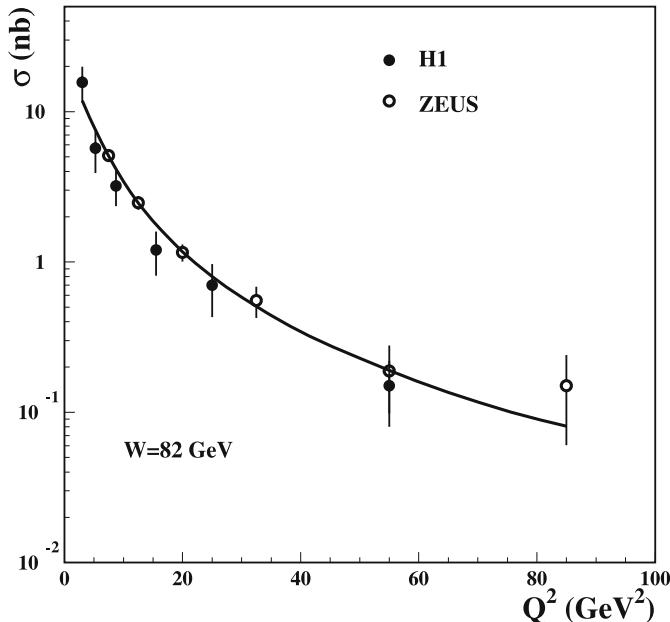
Note that the ZEUS data points, which were taken at  $W = 89$  GeV and at  $Q^2 = 9.6$  GeV $^2$ , have been rescaled to the H1 values of  $W = 82$  GeV and  $Q^2 = 8$  GeV $^2$  using the fitted  $W$ -dependence and  $Q^2$ -dependence of the DVCS cross section:  $\sigma_{\text{DVCS}} \propto W^{0.75}$  and  $\sigma_{\text{DVCS}} \propto 1/(Q^2)^{1.54}$  [31].

For the proton forward PDFs, which are required to evaluate  $Q_0^i$ , we used the LO CTEQ5L parameterization [40].

One can see from Fig. 1 that the absolute value and the  $Q^2$ -dependence of the total DVCS cross section is described very well. The agreement with the data at the highest values of  $Q^2$  would have been worse, if we had used the  $Q^2$ -independent slope  $B$ .

From Fig. 2 one can see that the absolute value and the  $W$ -dependence of the DVCS cross section is also reproduced rather well. However, one should note the slight discrepancy between the ZEUS and H1 data points at lower values of  $W$  and large experimental errors at the high end of  $W$ .

It is important to emphasize that our predictions for the total DVCS cross section were made using the parameterization of the GPD, which contains no free parameters (the role of  $Q_2^i$  and  $\beta$ , see (7), is negligible in H1 and ZEUS kinematics). It is very remarkable that the agreement with the data is so good.



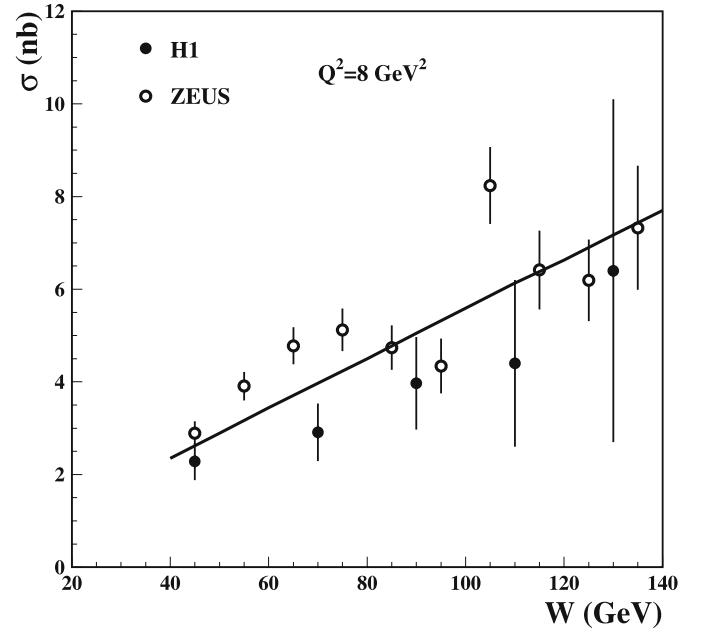
**Fig. 1.** The total DVCS cross section at  $W = 82$  GeV as a function of  $Q^2$ . The predictions of the dual parameterization of GPDs (solid curve) is compared to the H1 [32] and ZEUS [31]. The error bars represent the statistical and systematic uncertainties added in quadrature

In order to understand, at least partially, the success of the dual parameterization of GPDs in the description of the low- $x$  HERA data, it is instructive to analyze the DVCS amplitude  $\mathcal{A}_{\text{DVCS}}$  of (15) in some detail. Evaluating the imaginary and real parts of (15), one obtains [28]

$$\begin{aligned} \text{Im } \mathcal{A}(\xi, Q^2) &= \\ &- \sum_i e_i^2 \int_a^1 \frac{dx}{x} \frac{1}{\sqrt{2x/\xi - x^2 - 1}} \sum_{k=0}^2 x^k Q_k(x, 0, Q^2), \\ \text{Re } \mathcal{A}(\xi, t) &= \\ &- \sum_i e_i^2 \int_a^1 \frac{dx}{x} \sum_{k=0}^2 x^k Q_k(x, 0, Q^2) \\ &\times \left( \frac{1}{\sqrt{1+2x/\xi+x^2}} - 2\delta_{k0} \right) \\ &- \sum_i e_i^2 \int_0^a \frac{dx}{x} \sum_{k=0}^2 x^k Q_k(x, 0, Q^2) \\ &\times \left( \frac{1}{\sqrt{1-2x/\xi+x^2}} + \frac{1}{\sqrt{1+2x/\xi+x^2}} - 2\delta_{k0} \right), \end{aligned} \quad (17)$$

where  $a = (1 - \sqrt{1 - \xi^2})/\xi$ .

At low  $x_{Bj}$ ,  $\xi \approx x_{Bj}/2$  and the integration limit is  $a \approx \xi/2 = x_{Bj}/4$ . Thus, the functions  $Q_0^i$  and  $Q_2^i$  are never sampled at  $x < x_{Bj}/4$ , except for the second contribution to  $\text{Re } \mathcal{A}$ , see the last two lines of (17). However, this contribution is regular and small because the expression in



**Fig. 2.** The total DVCS cross section at  $Q^2 = 8$  GeV as a function of  $W$ . The predictions of the dual parameterization of GPDs (solid curve) is compared to the H1 [32] and ZEUS [31]. The error bars represent the statistical and systematic uncertainties added in quadrature

parenthesis behaves as  $x^2$  at small  $x$  for  $k = 0$ , which implies that the integrand behaves as  $x Q_{0,2}^i(x, 0, Q^2)$  at small  $x$ . This is clearly an advantage over the double distribution parameterization of GPDs, where, unless a special shape of the input PDF is assumed, the forward parton distributions are sampled all the way down to the unmeasurable  $x = 0$  [26].

In addition, the equations (17) are convenient for the numerical implementation since the integrands do not contain large end-point contributions, as can be explicitly seen by changing the integration variables.

## 4 Discussion and conclusions

We have presented and discussed the new leading order parameterization of GPDs introduced in [28]. In its minimal form, the parameterization is defined by the forward singlet quark PDFs and the form factors of the energy-momentum tensor, see (4) and (6). The  $t$ -dependence of the DVCS cross section was assumed in a simple factorized form with the  $Q^2$ -dependent slope, see (10) and (12).

We showed that our parameterization of GPDs describes the absolute value very well, the  $Q^2$ -dependence and  $W$ -dependence of HERA data on the total DVCS cross section. Moreover, since the data is at low  $x_{Bj}$ , our parameterization can be simplified by omitting the contribution of the  $Q_2^i$  function. This means that we have achieved a remarkably good description of a large set of the data on DVCS using a parameterization of GPDs, which contains no free parameters!

We discuss that our parameterization suits the low- $x_{Bj}$  kinematics especially well, because the quark singlet PDFs are never probed at the unmeasurably low values of Bjorken  $x$  and because the expression for the DVCS amplitude is numerically stable. This allows us to advertise our model as a better alternative to the popular double distribution parameterization of the GPDs, at least in the low- $\xi$  region.

The parameterization presented in this work can be readily generalized by including more  $Q_k^i$  functions, considering the GPDs  $E$ ,  $\tilde{H}$  and  $\tilde{E}$  and by using more elaborate models of the  $t$ -dependence. This was not necessary in H1 [32] and ZEUS [31] kinematics, but might be required for HERMES and CLAS kinematics.

Also, the role of next-to-leading order (NLO) corrections and higher twist effects should be investigated. In particular, it is important to compare the size of the NLO corrections using dual parameterization with the results of the analysis using double distribution parameterization, where the NLO corrections are found to be large [26].

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